

Quarks with Integer Electric Charge

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Abstract

Within the context of the Standard Model, quarks are placed in a $(\mathbf{3}, \mathbf{2}) \oplus (\mathbf{3}, \bar{\mathbf{2}})$ matter field representation of $U_{EW}(2)$. Although the quarks carry unit intrinsic electric charge in this construction, anomaly cancellation constrains the Lagrangian in such a way that the quarks' associated currents couple to the photon with the usual $2/3$ and $1/3$ fractional electric charge associated with conventional quarks. The resulting model is identical to the Standard Model in the $SU_C(3)$ sector: However, in the $U_{EW}(2)$ sector it is similar but not necessarily equivalent. Off hand, the model appears to be phenomenologically equivalent to the conventional quark model in the electroweak sector for experimental conditions that preclude observation of individual constituent currents. On the other hand, it is conceivable that detailed analyses for electroweak reactions may reveal discrepancies with the Standard Model in high energy and/or large momentum transfer reactions.

The possibility of quarks with integer electric charge strongly suggests the notion that leptons and quarks are merely different manifestations of the same underlying field. A speculative model is proposed in which a phase transition is assumed to occur between $SU_C(3) \otimes U_{EM}(1)$ and $U_{EM}(1)$ regimes. This immediately; explains the equality of lepton/quark generations and lepton/hadron electric charge, relates neutrino oscillations to quark flavor mixing, reduces the free parameters of the Standard Model, and renders the issue of quark confinement moot.

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I. INTRODUCTION

The basis of this paper is the realization that the intrinsic charges carried by a matter field and the coupling strengths of its associated currents are not necessarily equivalent. This realization was expounded in [1] where the quantum numbers—characterizing elementary fields—and gauge/matter field coupling strengths were analyzed for gauge theories with direct product groups.

As an illustrative example, consider a theory with internal symmetry $SU(3) \otimes SU(2) \otimes U(1)$ and suppose that the matter field Ψ characterized by the quantum numbers (C, I, Y) and its $SU(2) \otimes U(1)$ conjugate field Ψ' characterized by the quantum numbers $(C, \bar{I}, \bar{Y}) = (C, I, -Y)$ are elementary fields furnishing the representation $(\mathbf{3}, \mathbf{2}, \mathbf{1}) \oplus (\mathbf{3}, \bar{\mathbf{2}}, \bar{\mathbf{1}})$. The Lagrangian density will contain two terms of the form $\alpha^2(\bar{\Psi} \not{D} \Psi)$ and $\beta^2(\bar{\Psi}' \not{D}' \Psi')$ with $\alpha, \beta \in \mathbb{R}$ and $\alpha^2 + \beta^2 = 1$. In the absence of the $SU(3)$ symmetry, α^2 and β^2 are necessarily equal and can be absorbed into a redefinition of the matter fields. However, in some circumstances $\alpha^2 \neq \beta^2$ and a renormalization of the matter fields can not absorb the relative factor of α/β (since Ψ and Ψ' are $SU(2) \otimes U(1)$ conjugate).

It is clear that Ψ carries the $U(1)$ charge Y yet its associated current couples to the $U(1)$ gauge field with strength $\alpha^2 Y$. Likewise, Ψ' carries charge $-Y$ but its associated current couples with strength $-\beta^2 Y$. Note, however, that the $SU(3)$ ‘charges’ and coupling strengths are equivalent because $\alpha^2 + \beta^2 = 1$. Also note that this phenomenon does not occur if only the $(\mathbf{3}, \mathbf{2}, \mathbf{1})$ matter field is included.

To see how this can be applied to a specific model of hadronic constituents (HC), it is best to first recall the historical motivation leading to the conventional assignment of quark quantum numbers.

The Standard Model (SM) began as an electroweak theory of leptons [2, 3, 4]. Later, hadrons were incorporated by considering the known structure of the charged hadronic current, the postulated quark composition of hadrons, and the assumed isospin and hypercharge quark quantum numbers [5, 6, 7].

The canonical status enjoyed by the isospin and hypercharge quantum numbers of quarks can be attributed to the structure of the $SU_I(2) \otimes U_Y(1)$ symmetry (sub)group and the success of the Gell-Mann/Nishijima relation ($Q \propto T + 1/2Y$) in classifying mesons and baryons in various approximate isospin and flavor symmetry models. Historically, this led

to the conclusion that the (u, d, s) quarks possessed fractional electric charge. Including the $SU_C(3)$ symmetry in the SM, assuming fractional electric charge, and using the Gell-Mann/Nishijima relation leads naturally to the conventional assignment of quarks in the $(\mathbf{3}, \mathbf{2}, \mathbf{1}/\mathbf{3})$ representation of $SU_C(3) \otimes SU_I(2) \otimes U_Y(1)$.

Now, suspending momentarily our notion of quarks and their assumed isospin and hypercharge, imagine HC[21] corresponding to a matter field representation in the unbroken electroweak symmetry domain of the SM, viz. $SU_W(2) \otimes U_{EM}(1)$. A natural assignment for the HC is a $(\mathbf{2}, \mathbf{1})$ field and a $(\bar{\mathbf{2}}, \bar{\mathbf{1}}) = (\mathbf{2}, -\mathbf{1})$ anti-field. (By natural I mean that there exists a preferred basis in the Lie algebra in which the charged gauge bosons have an electric charge of $\pm e$; and one might expect the gauge bosons exchange this electric charge quanta with elementary matter fields.) Now assign the HC to an $SU_C(3)$ triplet without recourse to the Gell-Mann/Nishijima relation. Should the HC matter fields furnish a $(\mathbf{3}, \mathbf{2}, \mathbf{1})$, a $(\mathbf{3}, \mathbf{2}, -\mathbf{1})$, or a combination of the two? It is not unreasonable to expect a combination.

An apparent contradiction arises immediately: how can a color triplet of HC, which possess integer electric charge, combine to form hadrons with their observed electric charges? The answer is that the intrinsic electric charge carried by an elementary matter field and its associated coupling strength to a gauge boson are not necessarily equivalent by the mechanism explained above if both $(\mathbf{3}, \mathbf{2}, \mathbf{1})$ and $(\mathbf{3}, \mathbf{2}, -\mathbf{1})$ HC fields are included.

It turns out that an appropriate combination of $(\mathbf{3}, \mathbf{2}, \mathbf{1})$ and $(\mathbf{3}, \mathbf{2}, -\mathbf{1})$ HC matter terms can be implemented within the context of the SM Lagrangian, and anomaly cancellation uniquely determines the relative factors (α^2 and β^2) in the terms. (Henceforth, I will designate HC based on this new representation ‘iquarks’ in order to clearly differentiate between the new quarks with integer electric charge and conventional quarks.) Consequently, the iquark matter fields couple to the electroweak gauge bosons with fractional coupling strengths reminiscent of conventional quark couplings. Specifically, the electromagnetic current contains the expected $2/3e$ and $-1/3e$ factors even though the iquarks have integer intrinsic electric charge.

With this matter field representation, the usual predictions of the SM that do not depend on iquark electroweak currents, as well as anomaly cancellation and resolution of the $\pi^0 \rightarrow 2\gamma$ problem, are exactly maintained. The iquark electroweak currents can be cast in terms of conventional quarks by identifying the up and down quark with an iquark doublet and its electroweak anti-doublet within each generation. It follows that conventional quarks are,

in a sense, an average of two iquarks. Insofar as experiments are not able to distinguish individual iquark currents, it appears that the predictions of this model for the electroweak sector will coincide with those of the SM in terms of conventional quarks.

Despite superficial appearances, there is a possibility that the iquark electroweak sector could yield results that differ from the SM—especially at high energy where individual currents might be distinguished due to mass differences of the iquarks. However, without detailed analyses of reaction rates (which is not undertaken here), the question of whether electroweak predictions of this model differ from the SM predictions remains open.

The choice of iquark representation advocated here is interesting since it: (i) reproduces many (if not all) of the successful predictions of the SM, (ii) may lead to experimentally verifiable differences, and (iii) suggests a closer kinship between hadronic constituents and leptons. In fact, the close kinship leads to the conjecture that leptons and iquarks are different manifestations of the same matter field. The idea is that for certain regions of field phase space (ostensibly characterized by configurations with particle content depending on space-time separation and relative four-momentum) there is a phase change.

A relationship between leptons and iquarks would achieve an economy of elementary particles and free parameters as well as suggest new models for extensions of the SM. It is interesting to note that neutrino oscillations would imply flavor mixing in the iquark electroweak currents which leads to the hope of gleaning some relationship between matter field masses, QCD and the Kobayashi-Maskawa matrix.

It should be mentioned that quarks with integer charge have been proposed before (see, e.g., [5], [8], and the review of [9]). However, the symmetry groups of these models are not the $SU_C(3) \otimes SU_W(2) \otimes U_{EM}(1)$ of the SM, and the iquark model presented here is neither related to these models nor inspired by them. Also, the proposed iquarks are not “preons” or “pre-quarks” (see, e.g., [10, 11, 12] and the review of [13]). That is, conventional quarks are not composite states of the iquarks. Instead, within this framework, conventional quarks can be interpreted as an *average* description of the iquarks.

II. INTRINSIC CHARGE AND COUPLING STRENGTH

Before considering the specific model, it is helpful to examine the relationship between intrinsic charges and coupling strengths associated with an internal symmetry group.[22] The

special case under consideration is a gauge field theory with direct product group $K = G \times H$ where G and H are simple compact and/or $U(1)$ Lie groups. Let the associated Lie algebras, \mathcal{G} and \mathcal{H} respectively, be generated by the bases $\{\mathbf{g}_i\}_{i=1}^{\dim G}$ and $\{\mathbf{h}_r\}_{r=1}^{\dim H}$.

Suppose there exist matter fields that furnish inequivalent irreducible representations (irreps) $\rho^{(a)}(G)$ and $\rho^{(b)}(H)$. Then the inequivalent irreps of the direct product group $\rho^{(a \times b)}(GH) = \rho^{(a)}(G) \otimes \rho^{(b)}(H)$ are comprised of *all* combinations of a and b . The associated representations of the Lie algebra $\mathcal{K} = \mathcal{G} \oplus \mathcal{H}$ are $\rho_e^{(a \times b)'}(\mathbf{g}_i + \mathbf{h}_r) = \rho_e^{(a)'}(\mathbf{g}_i) \oplus \rho_e^{(b)'}(\mathbf{h}_r)$ where ρ_e' is the derivative map of the representation evaluated at the identity element. It is $\rho_e'(\mathcal{K})$ that determines the normalization of the gauge fields via an inner product and the gauge/matter field interactions via the covariant derivative \mathcal{D} .

Now, the scale of the intrinsic charges carried by the gauge fields is determined by an inner product on the Lie algebra. The scale ambiguity of the inner product for each simple compact and $U(1)$ subgroup contributes an adjustable parameter (that can be absorbed into the definition of the gauge field). The interaction of the gauge fields with the matter fields (as encoded in the (renormalized) covariant derivative) results in an exchange of these charge quanta, and this characterizes the intrinsic charges of the matter fields.

On the other hand, the coupling strengths between the gauge fields and matter currents are determined by the specific form of the matter field Lagrangian. The most general (spinor) matter field Lagrangian density consistent with the requisite symmetries consists of a sum over the inequivalent irreps of the direct product group:

$$\mathcal{L}_m = i \sum_{a,b} \kappa_{ab} \bar{\psi}^{(a \times b)} \cdot \mathcal{D} \psi^{(a \times b)} + \text{mass terms} \quad (2.1)$$

where κ_{ab} are positive real constants that are constrained by various consistency conditions.

These matter field terms give rise to the covariantly conserved matter field currents

$$j_{(i,r)}^\mu = \sum_{a,b} \kappa_{ab} \bar{\psi}^{(a \times b)} \cdot \gamma^\mu \rho_e^{(a \times b)'}(\mathbf{g}_i + \mathbf{h}_r) \psi^{(a \times b)} . \quad (2.2)$$

Evidently, the ratios of coupling strengths and associated intrinsic charges are given by the κ_{ab} . It is clear that κ_{ab} can be absorbed by a field redefinition if either (i) $G = I$ or $H = I$, or (ii) the matter fields are not related somehow. Otherwise, non-trivial κ_{ab} may persist.

III. HADRONIC CONSTITUENTS

Since the local details of the SM depend on the Lie algebra of the gauge group—insofar as the Lagrangian density is concerned—and since $su(2) \oplus u(1) \cong u(2)$, I may as well use $U(2) = SU(2) \otimes U(1)/Z_2$ instead of $SU(2) \otimes U(1)$ for the electroweak gauge group. Moreover, the emphasis on electric charge suggests using $U_{EW}(2)$, which is characterized by the Lie algebra decomposition $su_W(2) \oplus u_{EM}(1)$. The idea now is to have the iquark matter fields furnish the same $U_{EW}(2)$ representation as the lepton matter fields. Since the iquarks are to have integer charge, there must be some mechanism to effect fractional couplings in the electroweak currents. The solution is to consider a combination of a $(\mathbf{3}, \mathbf{2})$ and a $(\mathbf{3}, \overline{\mathbf{2}})$ of $SU_C(3) \otimes U_{EW}(2)$ [23]. Anomaly cancellation determines the allowed combination, and the necessary fractional couplings ensue.

Remark: There are good reasons to believe the electroweak group is $U(2)$. First, if ρ is a representation of $SU(2) \otimes U(1)$ furnished by the lepton fields of the SM, then $\ker \rho = Z_2$ and therefore the lepton matter fields do not furnish a faithful representation ([14]). The group that does act effectively on the matter fields is $SU(2) \otimes U(1)/Z_2 = U(2)$. (Recall that we require faithful representations.) Second, both $SU(2) \otimes U(1)$ and $U(2)$ have the same covering group $Gl(1, q)$. Representations of $Gl(1, q)$ will descend to representations of $SU(2) \otimes U(1)$ or $U(2)$ if the associated discrete factor groups are represented trivially (i.e., by the unit matrix). For $SU(2) \otimes U(1)$ this requirement implies no relationship between isospin and hypercharge, but for $U(2)$ it implies $n = T + 1/2Y$ with n integer (see [16]). Identifying n with electric charge renders the Gell-Mann/Nishijima relation (and electric charge quantization) a consequence of the group $U(2)$. Third, symmetry reduction from $U(2)$ to $U(1)$ is less constrained than reduction from $SU(2) \otimes U(1)$ ([14]). Fourth, from a fiber bundle point of view, the most general structure group for a matter field doublet defined on a paracompact base space is $U(2)$.

A. The Model

First and foremost, we require the iquark matter fields to furnish inequivalent faithful irreps of $SU_C(3) \times U_{EW}(2)$ for physical reasons and so that the results of [1] can be applied.

For the iquark matter field sector of the model, the general (classical) setup begins with a principal bundle with structure group $SU_C(3) \times U_{EW}(2)$ along with associated vector bundles over Minkowski space-time $V_{\mathbf{R}} \rightarrow M^4$ where \mathbf{R} designates the representation furnished by a particular matter field. The typical fiber of $V_{\mathbf{R}}$ will depend on \mathbf{R} . For example, for the $(\mathbf{3}, \mathbf{2}) \oplus (\mathbf{3}, \bar{\mathbf{2}})$ of $SU_C(3) \times U_{EW}(2)$ the typical fiber is $\mathbb{C}^3 \otimes (\mathbb{C}^2 \oplus \mathbb{C}^2)$. Here \mathbb{C}^3 carries the fundamental representation of $SU_C(3)$, and $\mathbb{C}^2 \oplus \mathbb{C}^2$ carries the $U_{EW}(2)$ fundamental representation and its conjugate representation. The internal degrees of freedom of elementary matter fields are (by definition) identified with a chosen basis of the typical fiber of $V_{\mathbf{R}}$. Local gauge symmetry allows for a consistent choice of basis at each space-time point.

Spinor matter fields require the product bundle $S \otimes V_{\mathbf{R}}$ where S is a spinor bundle over Minkowski space-time. For example, given a trivialization of $S \otimes V_{(\mathbf{3}, \mathbf{2})}$, let $\{\mathbf{e}_{\alpha A a}\} := \{\psi_{\alpha} \otimes \mathbf{e}_A \otimes \mathbf{e}_a\}$ be the chosen basis that spans the typical fiber $\mathbb{C}^4 \otimes \mathbb{C}^3 \otimes \mathbb{C}^2$. (Indices are assumed to have the necessary ranges for any given representation.) Sections $\Psi = \Psi^{\alpha A a} \mathbf{e}_{\alpha A a}$ of $S \otimes V_{(\mathbf{3}, \mathbf{2})}$ constitute the elementary spinor fields in the $(\mathbf{3}, \mathbf{2})$ representation, and $\mathbf{e}_A \otimes \mathbf{e}_a$ encode the internal $SU_C(3) \times U_{EW}(2)$ degrees of freedom. For the conjugate representation $(\mathbf{3}, \bar{\mathbf{2}})$, we have $\tilde{\Psi} = \Psi^{\alpha A \bar{a}} \mathbf{e}_{\alpha A \bar{a}} := [i\tau_2]_{\bar{a}}^a \Psi^{\alpha A a} (\psi_{\alpha} \otimes \mathbf{e}_A \otimes \mathbf{e}_a^*)$. There are analogous expressions for elementary fields furnishing the other representations.

The first step is to assign representations to the iquark matter fields. In order to simplify the presentation, Lorentz/spinor components of the fields will be suppressed since they are just treated in the usual manner. So attention will be restricted to the sub-bundles $V_{(\mathbf{3}, \mathbf{2})}$ and $V_{(\mathbf{3}, \mathbf{1})}$ over M^4 with structure group $SU_C(3) \times U_{EW}(2)$.

Analogy with lepton matter fields suggests defining the $(\mathbf{3}, \mathbf{2})$ iquark $\mathbf{H}^+ := H_+^A \mathbf{e}_A$ and its $U_{EW}(2)$ conjugate $(\mathbf{3}, \bar{\mathbf{2}})$ $\mathbf{H}^- := H_-^A \mathbf{e}_A$ by

$$H_+^A := \Psi^{Aa} \mathbf{e}_a = \Psi^{A1} \mathbf{e}_1 + \Psi^{A2} \mathbf{e}_2 =: (h^+ \mathbf{e}_1)^A + (\xi^0 \mathbf{e}_2)^A = \begin{pmatrix} h^+ \\ \xi^0 \end{pmatrix}^A \quad (3.1)$$

and

$$H_-^A := [i\tau_2]_{\bar{b}}^a \Psi^{A\bar{b}} \mathbf{e}_a^* = \Psi^{A2} \mathbf{e}_1^* - \Psi^{A1} \mathbf{e}_2^* =: \begin{pmatrix} \xi^0 \\ h^- \end{pmatrix}^A. \quad (3.2)$$

Here h^{\pm} and ξ^0 are complex space-time Dirac spinor fields (the superscripts denote electric charge) and $\mathbf{e}_{1,2}$ span \mathbb{C}^2 . The h^{\pm} and ξ^0 comprise the iquarks.[24] There are (at least)

three copies of \mathbf{H}^\pm accounting for the three iquark generations. No generality is sacrificed by assuming \mathbf{H}^\pm are normalized.

By assumption, both the left and right-handed iquarks furnish the $\mathbf{3}$ of $SU_C(3)$. The left-handed iquarks furnish the $\mathbf{2}$ and $\bar{\mathbf{2}}$ and the right-handed iquarks the $\mathbf{1}^+$, $\mathbf{1}^-$ and $\mathbf{1}^0$ of $U_{EW}(2)$. Thus, we have $\mathbf{H}_L^+ := (\mathbf{H}^+)_L$, \mathbf{H}_L^- , \mathbf{h}_R^+ , \mathbf{h}_R^- , and $\boldsymbol{\xi}_R^0$ furnishing the $(\mathbf{3}, \mathbf{2})$, $(\mathbf{3}, \bar{\mathbf{2}})$, $(\mathbf{3}, \mathbf{1}^+)$, $(\mathbf{3}, \mathbf{1}^-)$, and $(\mathbf{3}, \mathbf{1}^0)$ respectively.

Next, the gauge potential must be specified in the relevant representations. In the broken $U_{EW}(2)$ symmetry regime, which is characterized by matter fields with conserved electric charge, the Lie algebra $u_{EW}(2)$ decomposes as $u_{EM}(1) \oplus (u_{EW}(2)/u_{EM}(1))$. Thus the gauge bosons are also characterized by electric charge. This implies that the broken symmetry generators are eigenvectors of the adjoint map of the unbroken, electric charge generator. That is, the Lie algebra decomposition is $u_{EW}(2) = u_{EM}(1) \oplus \mathfrak{k}$ such that $u_{EM}(1) \cap \mathfrak{k} = 0$ and $ad(u_{EM}(1))\mathfrak{k} \subseteq \mathfrak{k}$. Since $U_{EW}(2)$ has rank 2, the relevant basis is

$$\begin{aligned} [\mathbf{e}_\pm, \mathbf{e}_\mp] &= \sum_i \pm c'_i \mathbf{h}_i, \\ [\mathbf{e}_\pm, \mathbf{h}_i] &= \pm c_i \mathbf{e}_\pm, \\ [\mathbf{h}_i, \mathbf{h}_j] &= 0, \end{aligned} \tag{3.3}$$

where $\{\mathbf{e}_+, \mathbf{e}_-, \mathbf{h}_1, \mathbf{h}_2\}$ spans $u_{EW}(2)$, and c_i, c'_i are constants with $i, j \in \{1, 2\}$. The most general 2-dimensional representation allowed by (3.3) is generated by

$$\begin{aligned} \mathbf{T}_+ &:= \rho'_e(\mathbf{e}_+) = i \begin{pmatrix} 0 & t \\ 0 & 0 \end{pmatrix}, \\ \mathbf{T}_- &:= \rho'_e(\mathbf{e}_-) = i \begin{pmatrix} 0 & 0 \\ t & 0 \end{pmatrix}, \\ \mathbf{T}_0 &:= \rho'_e(\mathbf{h}_1) = i \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix}, \\ \mathbf{Q} &:= \rho'_e(\mathbf{h}_2) = i \begin{pmatrix} r & 0 \\ 0 & s \end{pmatrix}, \end{aligned} \tag{3.4}$$

where r, s, t, u, v are real constants and $\rho : U_{EW}(2) \rightarrow GL(\mathbb{C}^2)$ (ρ'_e denotes the derivative of the representation map ρ evaluated at the identity element $e \in U_{EW}(2)$).

To proceed, an ad invariant positive definite inner product on $u_{EW}(2)$ is required. In fact, there is a 2-dimensional real vector space of positive definite ad invariant bilinear forms on

$u_{EW}(2)$ given by ([17])

$$-\langle \mathbf{t}_\alpha, \mathbf{t}_\beta \rangle = 2g_1^{-2} \text{Tr}(\mathbf{t}_\alpha \mathbf{t}_\beta) + (g_2^{-2} - g_1^{-2}) \text{Tr} \mathbf{t}_\alpha \cdot \text{Tr} \mathbf{t}_\beta \quad (3.5)$$

for $\mathbf{t}_\alpha, \mathbf{t}_\beta \in u_{EW}(2)$ where g_1 and g_2 are real parameters.[25] A positive definite inner product is obtained by the choice

$$g_{\alpha\beta} := (\mathbf{t}_\alpha, \mathbf{t}_\beta) \equiv -\langle \mathbf{t}_\alpha, \mathbf{t}_\beta \rangle . \quad (3.6)$$

Explicitly, in the basis defined by (3.3),

$$g_{\alpha\beta} = \begin{pmatrix} 0 & g_W^{-2} & 0 & 0 \\ g_W^{-2} & 0 & 0 & 0 \\ 0 & 0 & g_Z^{-2} & 0 \\ 0 & 0 & 0 & g_Q^{-2} \end{pmatrix} \quad (3.7)$$

where

$$g_W^{-2} := (\mathbf{e}_\pm, \mathbf{e}_\mp), \quad g_Z^{-2} := (\mathbf{h}_1, \mathbf{h}_1), \quad g_Q^{-2} := (\mathbf{h}_2, \mathbf{h}_2) . \quad (3.8)$$

The inner product can be put into canonical form by rescaling the $u_{EW}(2)$ basis vectors by $\mathbf{e}_\pm \rightarrow g_W \mathbf{e}_\pm$, $\mathbf{h}_1 \rightarrow g_Z \mathbf{h}_1$, and $\mathbf{h}_2 \rightarrow g_Q \mathbf{h}_2$.

Since the Lie algebra is a direct sum of semisimple and abelian algebras, the inner product defined on each Lie subalgebra is proportional to the inner product for any of its faithful representations. Hence, (3.1), (3.5), (3.8), and the orthogonality condition $u_{EM} \cap \mathfrak{k} = 0$ give the $u_{EW}(2)$ representation (superscript +) and the conjugate representation (superscript -)

for the doublet iquark matter fields;

$$\begin{aligned}
\mathbf{T}_0^+ &= \frac{ie}{2 \cos \theta_W \sin \theta_W} \begin{pmatrix} 2 \sin^2 \theta_W - 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
\mathbf{T}_0^- &= \frac{-ie}{2 \cos \theta_W \sin \theta_W} \begin{pmatrix} 1 & 0 \\ 0 & 2 \sin^2 \theta_W - 1 \end{pmatrix}, \\
\mathbf{T}_\pm^\pm &= \frac{ie}{\sqrt{2} \sin \theta_W} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \\
\mathbf{T}_\mp^\pm &= \frac{ie}{\sqrt{2} \sin \theta_W} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \\
\mathbf{Q}^+ &= ie \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \\
\mathbf{Q}^- &= -ie \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},
\end{aligned} \tag{3.9}$$

where e is the electric charge, θ_W is the Weinberg angle,

$$\begin{aligned}
g_Q^2 &= \frac{g_1^2 g_2^2}{(g_1^2 + g_2^2)} =: e^2 \\
g_W^2 &= \frac{g_1^2}{2} =: \frac{e^2}{2 \sin^2 \theta_W} \\
g_Z^2 &= \frac{(g_1^2 + g_2^2)}{4} = \frac{e^2}{4 \sin^2 \theta_W \cos^2 \theta_W}
\end{aligned} \tag{3.10}$$

and $r(s)$, t , and $v(u)$ were absorbed into g_Q, g_W , and g_Z . This is (not surprisingly) identical to the SM left-handed lepton representation.

The 1-dimensional representation for the right-handed iquarks is obtained by taking the trace of the 2-dimensional representation and using (3.3). For h^\pm it amounts to taking the trace of eq. (3.9). For the electrically neutral ξ^0 , it yields the trivial representation.

According to the discussion in Section II, the inequivalent irreps of the direct product group include the combinations $(\mathbf{3}, \mathbf{2})$, $(\mathbf{3}, \bar{\mathbf{2}})$, $(\mathbf{3}, \mathbf{1}^+)$, $(\mathbf{3}, \mathbf{1}^-)$, and $(\mathbf{3}, \mathbf{1}^0)$ along with the corresponding anti-particle combinations. We postulate that the iquark matter field part of the Lagrangian density is comprised of a sum over these combinations with appropriate

weights. The iquark contribution to the Lagrangian density is therefore

$$\begin{aligned} \mathcal{L}_{\text{iquark}} = & i \sum_s \kappa^+ (\overline{\mathbf{H}}_{\text{L},s}^+ \not{D}^+ \mathbf{H}_{\text{L},s}^+ + \overline{\mathbf{H}}_{\text{R},s}^+ \not{D}^+ \mathbf{H}_{\text{R},s}^+) \\ & + \kappa^- (\overline{\mathbf{H}}_{\text{L},s}^- \not{D}^- \mathbf{H}_{\text{L},s}^- + \overline{\mathbf{H}}_{\text{R},s}^- \not{D}^- \mathbf{H}_{\text{R},s}^-) \end{aligned} \quad (3.11)$$

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = & - \sum_{s,t} m_{st} \overline{\mathbf{H}}_{\text{L},s}^+ \Phi^- \mathbf{h}_{\text{R},t}^+ + n_{st} \overline{\mathbf{H}}_{\text{L},s}^+ \Phi^+ \boldsymbol{\xi}_{\text{R},t}^0 \\ & + H.c. \end{aligned} \quad (3.12)$$

where s, t label iquark generation. The matrices m_{st} and n_{st} are general, generation(flavor) mixing mass matrices, and Φ^+ is Higgs field

$$\Phi^+ := \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \quad (3.13)$$

The covariant derivatives are

$$\not{D}^+ \mathbf{H}_{\text{L}}^+ = (\not{\partial} + \mathcal{W}^+ \mathbf{T}_+^+ + \mathcal{W}^- \mathbf{T}_-^+ + \mathcal{Z}^0 \mathbf{T}_0^+ + \mathcal{A} \mathbf{Q}^+ + \mathcal{G} \boldsymbol{\Lambda}) \mathbf{H}_{\text{L}}^+$$

$$\not{D}^- \mathbf{H}_{\text{L}}^- = (\not{\partial} + \mathcal{W}^{+*} \mathbf{T}_+^- + \mathcal{W}^{-*} \mathbf{T}_-^- + \mathcal{Z}^0 \mathbf{T}_0^- + \mathcal{A} \mathbf{Q}^- + \mathcal{G} \boldsymbol{\Lambda}) \mathbf{H}_{\text{L}}^-$$

$$\not{D}^\pm \mathbf{h}_{\text{R}}^\pm := \text{tr}[\not{D}^\pm] \mathbf{h}_{\text{R}}^\pm,$$

$$\not{D} \boldsymbol{\xi}_{\text{R}}^0 = (\not{\partial} + i \mathcal{G} \boldsymbol{\Lambda}) \boldsymbol{\xi}_{\text{R}}^0, \quad (3.14)$$

where the trace is only over $U_{EW}(2)$ indices, and we have not specified the representation $\boldsymbol{\Lambda}$ of $SU_C(3)$ since it will not be needed.

The Yang-Mills, lepton and Higgs contributions to the Lagrangian density are identical to the SM.

A few remarks are in order.

- Re-scaling the iquark fields cannot cancel the relative scale difference between \mathbf{H}^+ and \mathbf{H}^- since they are $U_{EW}(2)$ conjugate to each other (unless $\kappa^+ = \kappa^-$). Consequently, these factors are not trivial and their ratio is not altered by renormalization. The effect of the constants κ^+ and κ^- is to re-scale the charge e in (3.9). Note that

normalization of the $SU_C(3)$ coupling strengths is not altered as long as $\kappa^+ + \kappa^- = 1$, That is, $\kappa^+ + \kappa^- = 1$ guarantees the $SU_C(3)$ intrinsic charge and coupling strength equality (see [1]).

- The κ^+ and κ^- terms in $\mathcal{L}_{\text{iquark}}$ are not invariant under distinct $U(2)$; again because \mathbf{H}^+ and \mathbf{H}^- are $U_{EW}(2)$ conjugate to each other.
- The ξ_R^0 fields completely decouple from the $U_{EW}(2)$ gauge bosons. However, they do couple to the $SU_C(3)$ gauge bosons. They also have an induced mass due to the Higgs interaction.

B. Currents and Anomalies

Using (3.1), (3.9) and (3.11), the $U_{EW}(2)$ currents for each iquark generation work out to be[26]

$$\begin{aligned}
j_\mu^{0(Z)} &= \frac{e}{2 \sin \theta_W \cos \theta_W} \left[\kappa^+ \left(2 \sin^2 \theta_W - 1 \right) \overline{h_L^+} \gamma_\mu h_L^+ \right. \\
&\quad \left. - \kappa^- \left(2 \sin^2 \theta_W - 1 \right) \overline{h_L^-} \gamma_\mu h_L^- \right. \\
&\quad \left. + \kappa^+ 2 \sin^2 \theta_W \overline{h_R^+} \gamma_\mu h_R^+ + \kappa^+ \overline{\xi_L^0} \gamma_\mu \xi_L^0 \right. \\
&\quad \left. - \kappa^- \overline{\xi_L^0} \gamma_\mu \xi_L^0 - \kappa^- 2 \sin^2 \theta_W \overline{h_R^-} \gamma_\mu h_R^- \right], \\
j_\mu^{0(A)} &= \kappa^+ e \overline{h^+} \gamma_\mu h^+ - \kappa^- e \overline{h^-} \gamma_\mu h^-, \\
j_\mu^+ &= \frac{e}{\sqrt{2} \sin \theta_W} \left[\kappa^+ \overline{h_L^+} \gamma_\mu \xi_L^0 + \kappa^- \overline{\xi_L^0} \gamma_\mu h_L^- \right], \\
j_\mu^- &= \frac{e}{\sqrt{2} \sin \theta_W} \left[\kappa^+ \overline{\xi_L^0} \gamma_\mu h_L^+ + \kappa^- \overline{h_L^-} \gamma_\mu \xi_L^0 \right]
\end{aligned} \tag{3.15}$$

where we used $W^{+*} = W^-$ for the κ^- terms in the two charged currents and summation over color indices is implicit.

As is well known, for a consistent quantum version of this model to exist, the anomalies associated with these currents must cancel the lepton anomalies. Because the iquarks furnish the same representation as the leptons and because there are three color copies of each, one would not expect the anomalies in this model to cancel trivially.

To check this, it is possible to use an isospin/hypercharge basis in $U_{EW}(2)$. However, it is more direct to maintain the basis in which the unbroken $U(1)$ is associated with electric charge. It must be kept in mind that the $U_{EM}(1)$ quantities which enter into the anomaly calculation are not the intrinsic electric charges of the matter fields, per se, but the coupling strengths in the photon/matter field current, $j_\mu^{0(A)}$. The $U_{EM}(1)$, $SU_W(2)$ and $SU_C(3)$ contributions of the left-handed matter fields are given in Table I.

fermions	$(h^+, \xi^0)_L$	$(\xi^0, h^-)_L$	$\overline{h_R^+}$	$\overline{h_R^-}$	$\overline{\xi_R^0}$	$(\nu^0, l^-)_L$	$\overline{l_R^-}$
$U(1)$	$(\kappa^+, 0)$	$(0, -\kappa^-)$	$-\kappa^+$	κ^-	0	$(0, -1)$	1
$SU(2)$	2	2	1	1	1	2	1
$SU(3)$	3	3	$\overline{3}$	$\overline{3}$	$\overline{3}$	1	1

TABLE I: Anomaly contributions for left-handed fermionic matter fields.

There are only four cases to check including the gravitational anomaly ([19]): $[SU(2)]^2 U(1)$, $[SU(3)]^2 U(1)$, $[U(1)]^3$, and $[G]^2 U(1)$. In that order, the relevant terms are

$$\sum_{\text{doublets}} p = 3(\kappa^+) + 3(-\kappa^-) + (-1) = 0, \quad (3.16a)$$

$$\sum_{\text{triplets}} p = (\kappa^+) + (-\kappa^-) + (-\kappa^+) + (\kappa^-) + 0 = 0, \quad (3.16b)$$

$$\begin{aligned} \sum_{\text{all}} p^3 &= 3(\kappa^+)^3 + 3(-\kappa^-)^3 + 3(-\kappa^+)^3 + 3(\kappa^-)^3 \\ &\quad + 3(0)^3 + (-1)^3 + (1)^3 = 0, \end{aligned} \quad (3.16c)$$

$$\begin{aligned} \sum_{\text{all}} p &= 3(\kappa^+) + 3(-\kappa^-) + 3(-\kappa^+) + 3(\kappa^-) \\ &\quad + 3(0) + (-1) + (1) = 0, \end{aligned} \quad (3.16d)$$

where ep denotes the $U_{EM}(1)$ coupling parameter for the quark currents. With the exception of (3.16a), the anomaly conditions are null rather trivially. From (3.16a) and the condition $\kappa^+ + \kappa^- = 1$, there will be no anomaly associated with the gauge symmetries for the choice

$$\kappa^+ = \frac{2}{3}, \quad \kappa^- = \frac{1}{3}. \quad (3.17)$$

Now turn to the issue of the global chiral transformation,

$$\delta_\lambda h^+ = i\lambda\gamma_5 h^+ \quad \delta_\lambda \xi^0 = -i\lambda\gamma_5 \xi^0, \quad (3.18)$$

and the decay rate of $\pi^0 \rightarrow 2\gamma$. The chiral anomaly is proportional to

$$\text{tr} \left[(\kappa^+ \mathbf{Q}^+ + \kappa^- \mathbf{Q}^-)^2 \tau_3 \right] \quad (3.19)$$

which, for three colors, yields

$$3 \left(\frac{2}{3} \right)^2 - 3 \left(\frac{1}{3} \right)^2 = 1 . \quad (3.20)$$

Not surprisingly, this is identical to the result of the SM and yields the correct decay rate.

To make contact with the SM, the conventional SM (generation-mixed) quark mass eigenstates can be associated with h^\pm and ξ . Inspection of (3.15) suggests that the familiar fractionally charged quarks should be associated with a pair of fields. Thus, make the following (one-way) correspondence:

$$u^{+\frac{2}{3}} \sim (h^+, \xi^0) \quad (3.21a)$$

$$d^{-\frac{1}{3}} \sim (\xi^0, h^-) \quad (3.21b)$$

where u and d represent up and down quark fields respectively. More accurately, the quark currents are identified with a pair of h^\pm and ξ^0 currents:

$$\begin{aligned} \overline{u_L^{+\frac{2}{3}}} \gamma_\mu u_L^{+\frac{2}{3}} &\sim \left(\overline{h_L^+} \gamma_\mu h_L^+, \overline{\xi_L^0} \gamma_\mu \xi_L^0 \right) \\ \overline{d_L^{-\frac{1}{3}}} \gamma_\mu d_L^{-\frac{1}{3}} &\sim \left(\overline{\xi_L^0} \gamma_\mu \xi_L^0, \overline{h_L^-} \gamma_\mu h_L^- \right) \\ \overline{u_L^{+\frac{2}{3}}} \gamma_\mu d_L^{-\frac{1}{3}} &\sim \left(\overline{h_L^+} \gamma_\mu \xi_L^0, \overline{\xi_L^0} \gamma_\mu h_L^- \right) . \end{aligned} \quad (3.22)$$

So that, for example,

$$\left(\frac{4}{3} \sin^2 \theta - 1 \right) \overline{u_L^{+\frac{2}{3}}} \gamma_\mu u_L^{+\frac{2}{3}} \sim \left(\frac{4}{3} \sin^2 \theta - \frac{2}{3} \right) \overline{h_L^+} \gamma_\mu h_L^+ - \frac{1}{3} \overline{\xi_L^0} \gamma_\mu \xi_L^0 , \quad (3.23)$$

and

$$\overline{u_L^{+\frac{2}{3}}} \gamma_\mu d_L^{-\frac{1}{3}} \sim \frac{2}{3} \overline{h_L^+} \gamma_\mu \xi_L^0 + \frac{1}{3} \overline{\xi_L^0} \gamma_\mu h_L^- . \quad (3.24)$$

There are analogous relations for the currents $\overline{d_L^{-\frac{1}{3}}} \gamma_\mu d_L^{-\frac{1}{3}}$ and $\overline{d_L^{-\frac{1}{3}}} \gamma_\mu u_L^{+\frac{2}{3}}$. These correspondences will certainly lead to iquark current masses and hadronic constituents that differ from the conventional quark picture. Appendix A contains some *tentative* iquark composites for a selection of mesons and baryons.

To the extent that this correspondence is justified, the weak currents in (3.15) agree with the SM currents. Graphically, the correspondence associates a sum of one-particle currents

and their vertex factors with an equivalent two-particle current, whose vertex factor is the sum of the individual vertex factors of the constituent one-particle currents. Physically, the correspondence constitutes an average description in the sense that individual iquarks are not discriminated.

Although the electromagnetic current couples to the iquarks with the correct fractional charge, it does not couple to all of the energy-momentum carried by the iquarks since ξ^0 is electrically neutral. However, as discussed later, it is expected that $m_h/m_\xi \approx m_l/m_\nu$. So if this affects transition probabilities, it would presumably be a small effect.

Evidently, the electroweak currents in (3.15) will agree with the SM whenever: (i) individual quarks/iquarks cannot be observed and (ii) the experiment is not sensitive to the (assumed) relatively small mass of ξ . At this point in time, the first case is ruled out. However, it is conceivable that some types of experiments could be sensitive to the small mass ratio m_ξ/m_h . This might[27] lead to disagreement with the SM that would presumably become more prominent at high energy and large momentum transfer.

IV. A SPECULATIVE MODEL

Even if it turns out that the iquark model cannot be experimentally distinguished from the SM, it is theoretically different because the leptons and iquarks furnish the same $U_{EW}(2)$ representation. This fact begs the conjecture that leptons and iquarks are just different manifestations of the same underlying field.

Conventional wisdom is that $SU_C(3) \times U_{EM}(1)$ is unbroken throughout the entire phase space. However, this assumes that leptons and iquarks are separate matter fields. It is conceivable that matter fields exhibit different symmetry characteristics dependent on space-time position, four-momentum, and particle content. Perhaps leptons and iquarks are different phases of the same matter field and that $SU_C(3)$ is broken in the lepton phase.

Such a phase change would have to depend not only on the QCD characteristic energy but also on the localized particle content. Presumably then, the $SU(3)$ symmetry would require not only sufficient energy but also the necessary iquark particle content sufficiently localized in space-time. This, together with asymptotic freedom, would conspire to suppress strong interactions in typical lepton-hadron collisions.

It is interesting to implement this idea in terms of an effective Lagrangian density—local

in configuration space. In this speculative model, the Yang-Mills and Higgs contributions to the Lagrangian density will be identical to the SM so I will not bother to display them. Also, the Yukawa term is taken to be the same as in (3.11) except there will be obvious adjustments to the mass matrices. However, in place of the usual quark and lepton contributions, there will be a single contribution denoted \mathcal{L}_f . The relevant term is

$$\mathcal{L}_f = i \sum_s \bar{\mathbf{F}}_s \not{D} \mathbf{F}_s \quad (4.1)$$

where

$$\mathbf{F} := \mathbf{S} \begin{pmatrix} \mathbf{0} \\ \mathbf{F}^- \end{pmatrix}, \quad \begin{pmatrix} \mathbf{F}^+ \\ \mathbf{F}^- \end{pmatrix} \in \mathbb{C}^2 \oplus \mathbb{C}^2, \quad (4.2)$$

$$\mathbf{S} = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}, \quad \mathbf{S} \mathbf{S}^T = \mathbf{1}, \quad (4.3)$$

$$\not{D} := \begin{pmatrix} \not{D}^+ & 0 \\ 0 & \not{D}^- \end{pmatrix}. \quad (4.4)$$

\not{D} acts in the usual (asymmetrical) way on left/right-handed fields, and includes $SU_C(3) \times U_{EW}(2)$ gauge fields.

The matter fields \mathbf{F} are sections of $S \otimes V_f$ where V_f is the Whitney sum bundle of the vector bundles associated with the representations furnished by the left/right-handed \mathbf{F} . As in previous sections, \mathbf{F}^+ and \mathbf{F}^- share the same space-time dependence and spinor and color indices. Clearly (4.1) reduces to

$$\mathcal{L}_f = i \sum_s \beta^2 \overline{\mathbf{F}^+} \not{D}^+ \mathbf{F}^+ + \alpha^2 \overline{\mathbf{F}^-} \not{D}^- \mathbf{F}^-. \quad (4.5)$$

If there is a phase transition—either induced by terms already present in (4.1) or in an added term—for some regions of phase space, then the matter field phase space \mathcal{P} will have the form $\mathcal{P} = \mathcal{P}_{SU_C(3) \times U_{EM}(1)} \cup \mathcal{P}_{U_{EM}(1)}$ with $\mathcal{P}_{SU_C(3) \times U_{EM}(1)} \cap \mathcal{P}_{U_{EM}(1)} = \emptyset$ where $\mathcal{P}_{SU_C(3) \times U_{EM}(1)}$ and $\mathcal{P}_{U_{EM}(1)}$ represent field and canonical conjugate field configurations of unbroken and broken $SU_C(3)$ respectively. In consequence, a functional integral over \mathcal{P} breaks into a sum of functional integrals over $\mathcal{P}_{SU_C(3) \times U_{EM}(1)}$ and $\mathcal{P}_{U_{EM}(1)}$.

Since the gauge field associated with $U_{EM}(1)$ exists (presumably) continuously throughout \mathcal{P} , the conditions of anomaly cancellation must hold everywhere in phase space. In other words, photon exchange is possible for all charged matter fields so $U_{EM}(1)$ currents must

match across boundaries of $\mathcal{P}_{SU_C(3) \times U_{EM}(1)}$ and $\mathcal{P}_{U_{EM}(1)}$. Therefore, as a phase change occurs, anomaly cancellation requires that \mathbf{S} changes discontinuously from $\alpha = 1, \beta = 0$ in $\mathcal{P}_{U_{EM}(1)}$ to $\alpha = \sqrt{1/3}, \beta = \sqrt{2/3}$ in $\mathcal{P}_{SU_C(3) \times U_{EM}(1)}$.

The theory can be conveniently reformulated in terms of a functional integral over the full phase space \mathcal{P} with full $SU_C(3) \times U_{EW}(2)$ symmetry by introducing separate iquark (h^\pm, ξ^0) and lepton (l^\pm, ν^0) fields having compact support on $\mathcal{P}_{SU_C(3) \times U_{EM}(1)}$ and $\mathcal{P}_{U_{EM}(1)}$ respectively. In this case, \mathcal{L}_f reduces to an effective $\mathcal{L}_{\text{iquark}} + \mathcal{L}_{\text{lepton}}$ with their associated α and β values.

In any case, for a state evolving from a region $\mathcal{P}_{SU_C(3) \times U_{EM}(1)} \leftrightarrow \mathcal{P}_{U_{EM}(1)}$, it follows that

$$\begin{pmatrix} h^\pm \\ \xi^0 \end{pmatrix}_s \longleftrightarrow \begin{pmatrix} l^\pm \\ \nu^0 \end{pmatrix}_s \quad (4.6)$$

implying: (i) a massive neutrino whose right-handed component completely decouples (except for gravity), (ii) the equivalence of lepton and baryon electric charge, (iii) equal numbers of lepton and iquark generations, (iv) $m_{h_s}/m_{\xi_s} \approx m_{l_s}/m_{\nu_s}$ (ignoring renormalization effects), and (v) a relationship between neutrino oscillations and iquark flavor mixing. Additionally, it reduces the number of free parameters and renders quark confinement a non-issue (or rather transmutes it into a deconfinement issue).

The Yukawa term would imply that the bare lepton and iquark masses are identical. However, renormalization due to self-energy contributions destroys the degeneracy, and a (very) rough estimate using $\alpha_s(m_Z)/\alpha_e(m_Z) \approx 10^1$ suggests $m_{h_s} \approx 10m_{l_s}$.

V. SUMMARY

I have presented an alternative representation for quarks in the Standard Model. Central to the motivation is the idea that, within a given representation, elementary particle and antiparticle states with multiple charges associated with local internal symmetries should realize all possible charge combinations. This leads to the possibility of non-trivial factors multiplying certain matter field terms in the Lagrangian density.

Implementing this idea within the context of the Standard Model leads to iquarks with integer electric charge that, nevertheless, couple to the photon with fractional charges. The resulting model differs from the Standard Model, because some of the iquarks (with small

mass) do not couple to the photon. However, it is not clear if the difference is experimentally detectible.

The fact that the iquarks and leptons furnish the same $U_{EW}(2)$ representation suggests that they are manifestations of the same underlying field. This would seem to require a phase change in certain regions of field phase space. If this characterization turns out to be correct, it would relate some of the parameters of the SM, and, hopefully, aid in the search for an underlying theory.

APPENDIX A: MESONS AND BARYONS

The mesons and baryons will be composites of $\mathbf{H}_s \overline{\mathbf{H}}_t$ and $\mathbf{H}_s \mathbf{H}_t \mathbf{H}_u$ respectively where s, t, u label iquark generation. Although each component of $\mathbf{H} := (\mathbf{H}^+, \mathbf{H}^-)$ is a spinor, I will omit the various combinations of spin for simplicity. Since \mathbf{H}^\pm has two (electroweak) components, a little work is required to exhibit the elementary field content of the composites.

Table II contains *tentative* pseudoscalar assignments for the first two generations of composites $\mathbf{H}_1^\pm \overline{\mathbf{H}}_1^\pm$, $\mathbf{H}_1^\pm \overline{\mathbf{H}}_2^\pm$, and $\mathbf{H}_2^\pm \overline{\mathbf{H}}_2^\pm$. Denote the top component of \mathbf{H} by \wedge and the bottom component by \vee . Meson composites should correspond to combinations of $|\wedge\overline{\wedge}\rangle$, $|\vee\overline{\vee}\rangle$, and $\frac{1}{\sqrt{2}}|\wedge\overline{\vee} + \vee\overline{\wedge}\rangle$, (ignoring spin combinations). There, of course, will be many more possible combinations and admixtures of spin and generation than those represented in the table.

Table III contains assignments of selected spin 1/2 and 3/2 baryons to iquark composites $\mathbf{H}_1^\pm \mathbf{H}_1^\pm \mathbf{H}_1^\pm$, $\mathbf{H}_1^\pm \mathbf{H}_1^\pm \mathbf{H}_2^\pm$, $\mathbf{H}_1^\pm \mathbf{H}_2^\pm \mathbf{H}_2^\pm$, and $\mathbf{H}_2^\pm \mathbf{H}_2^\pm \mathbf{H}_2^\pm$. For simplicity, the table displays only the iquark doublet content.

It should be emphasized that the composite assignments in the tables are only tentative.

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- [21] I use the term hadronic constituents, denoted by HC, in order to keep a clear distinction from the standard quarks.
- [22] This section is a condensed version of the general analysis of [1].
- [23] This combination was initially proposed in [15].
- [24] The assignment $h^- \mathbf{e}_2 := -h^+ \mathbf{e}_2^*$ follows from the representation $\mathbf{Q}^-(h^- \mathbf{e}_2) = -h^- \mathbf{e}_2$ of the electric charge generator on $V_{(\mathbf{3}, \overline{2})}$ (see eq. (3.9)).
- [25] That this bilinear form is negative definite follows from $2g_1^{-2} \text{Tr}(\mathbf{t}_\alpha \mathbf{t}_\beta) + (g_2^{-2} - g_1^{-2}) \text{Tr} \mathbf{t}_\alpha \cdot \text{Tr} \mathbf{t}_\beta \leq 2g_1^{-2} \text{Tr}(\mathbf{t}_\alpha \mathbf{t}_\beta) + (g_2^{-2} - g_1^{-2}) \text{Tr}(\mathbf{t}_\alpha \mathbf{t}_\beta) = (g_1^{-2} + g_2^{-2}) \text{Tr}(\mathbf{t}_\alpha \mathbf{t}_\beta) < 0$.

- [26] For clarity, the notation will not indicate generation mixing. However, from now on, all $iquark$ fields are understood to be generation-mixed mass eigenstates; it being understood that a concomitant Kobayashi-Maskawa type matrix must now be included in \mathcal{L}_{iquark} .
- [27] To be sure, this model would require different parton distribution functions than the SM, but it is not clear if these would imply contradicting predictions for scattering cross sections.

Meson	doublet composite	iquark composition
π^0	$H_1^\pm \overline{H_1^\pm}$	$\sim (h_1^+ \overline{\xi_1^0} + \xi_1^0 \overline{h_1^+}) + \text{C.}$
π^+	$H_1^+ \overline{H_1^-}$	$\sim (h_1^+ \overline{\xi_1^0} + \xi_1^0 \overline{h_1^-})$
π^-	$H_1^- \overline{H_1^+}$	$\sim (h_1^- \overline{\xi_1^0} + \xi_1^0 \overline{h_1^+})$
$\left. \begin{matrix} K^0 \\ D^0 \end{matrix} \right\}$	$H_1^\pm \overline{H_2^\pm} + 1 \leftrightarrow 2$	$\sim \begin{cases} (h_1^+ \overline{\xi_2^0} + \xi_2^0 \overline{h_1^+}) + \text{C.} \\ (h_2^+ \overline{\xi_1^0} + \xi_1^0 \overline{h_2^+}) + \text{C.} \end{cases}$
$\left. \begin{matrix} K^+ \\ D^+ \end{matrix} \right\}$	$H_1^+ \overline{H_2^-} + 1 \leftrightarrow 2$	$\sim \begin{cases} (h_1^+ \overline{\xi_2^0} + \xi_2^0 \overline{h_1^-}) \\ (h_2^+ \overline{\xi_1^0} + \xi_1^0 \overline{h_2^-}) \end{cases}$
$\left. \begin{matrix} K^- \\ D^- \end{matrix} \right\}$	$H_1^- \overline{H_2^+} + 1 \leftrightarrow 2$	$\sim \begin{cases} (h_1^- \overline{\xi_2^0} + \xi_2^0 \overline{h_1^+}) \\ (h_2^- \overline{\xi_1^0} + \xi_1^0 \overline{h_2^+}) \end{cases}$
D_s^*	$H_2^\pm \overline{H_2^\pm}$	$\sim (h_2^+ \overline{\xi_2^0} + \xi_2^0 \overline{h_2^+}) + \text{C.}$
D_s^+	$H_2^+ \overline{H_2^-}$	$\sim (h_2^+ \overline{\xi_2^0} + \xi_2^0 \overline{h_2^-})$
D_s^-	$H_2^- \overline{H_2^+}$	$\sim (h_2^- \overline{\xi_2^0} + \xi_2^0 \overline{h_2^+})$

TABLE II: Iquark assignments for selected mesons. For the iquarks, the overline denotes an antifield, \pm and 0 denote electric charge, the subscript denotes the iquark generation, and C. stands for $U_{EW}(2)$ conjugate. Summation over color indices is implied.

Baryon	doublet composite
p	$\mathbf{H}_1^+ \mathbf{H}_1^+ \mathbf{H}_1^-$
n	$\mathbf{H}_1^+ \mathbf{H}_1^- \mathbf{H}_1^-$
Δ^-	$\mathbf{H}_1^- \mathbf{H}_1^- \mathbf{H}_1^-$
Δ^{++}	$\mathbf{H}_1^+ \mathbf{H}_1^+ \mathbf{H}_1^+$
Σ^+	$\mathbf{H}_1^+ \mathbf{H}_1^+ \mathbf{H}_2^-$
Σ^0, Λ	$\mathbf{H}_1^+ \mathbf{H}_1^- \mathbf{H}_2^-$
Σ^-	$\mathbf{H}_1^- \mathbf{H}_1^- \mathbf{H}_2^-$
Ξ^0	$\mathbf{H}_1^+ \mathbf{H}_2^- \mathbf{H}_2^-$
Ξ^-	$\mathbf{H}_1^- \mathbf{H}_2^- \mathbf{H}_2^-$
Σ_c^{++}	$\mathbf{H}_1^+ \mathbf{H}_1^+ \mathbf{H}_2^+$
Σ_c^+, Λ_c^+	$\mathbf{H}_1^+ \mathbf{H}_1^- \mathbf{H}_2^+$
Σ_c^0	$\mathbf{H}_1^- \mathbf{H}_1^- \mathbf{H}_2^+$
Ξ_c^+	$\mathbf{H}_1^+ \mathbf{H}_2^- \mathbf{H}_2^+$
Ξ_c^0	$\mathbf{H}_1^- \mathbf{H}_2^- \mathbf{H}_2^+$
Ω_c^0	$\mathbf{H}_2^- \mathbf{H}_2^- \mathbf{H}_2^+$
Ξ_{cc}^{++}	$\mathbf{H}_1^+ \mathbf{H}_2^+ \mathbf{H}_2^+$
Ξ_{cc}^+	$\mathbf{H}_1^- \mathbf{H}_2^+ \mathbf{H}_2^+$
Ω^-	$\mathbf{H}_2^- \mathbf{H}_2^- \mathbf{H}_2^-$
Ω_{cc}^+	$\mathbf{H}_2^- \mathbf{H}_2^+ \mathbf{H}_2^+$
Ω_{ccc}^{++}	$\mathbf{H}_2^+ \mathbf{H}_2^+ \mathbf{H}_2^+$

TABLE III: Iquark doublet assignments for selected spin 1/2 and 3/2 baryons.